

**MATHEMATICS: SPECIALIST 1 & 2**

**EXTENDED PIECE OF WORK 3**

**PART B**

Time: 60 minutes Total Marks 41

A reminder that Proof by Induction involves four steps, and these should be shown for each question.

Step 1 Show it is true for n = 1

Step 2 Assume it is true for n = k

Step 3 Prove it is true for n = k + 1

Step 4 Concluding statement

**1.** [5 marks]

Complete the following proof by induction.

Prove that n3 > n2 for n ≥ 2

Step 1

Step 2 Assume that k3 > k2 for k ≥ 2

Step 3 Show that (k + 1)3 > (k + 1)2 for k ≥ 2

k3 + 3k2 + 3k + 1 > . . . . . . . . . . . . . .

. . . . . . . . . . . . . . > 0

k( . . . . . . . . . . . . .) > 0

k( . . . . . . . . . . )2 > 0

Which is always true when k

⇒ the statement is true for n = k + 1 if it is true for n = k

Step 4 The statement is true for n = 2 so it must be true for n = 3

The statement is true for n = 3 so it must be true for n = 4, etc

Hence k3 > k2 for k ≥ 2

**2.** [5 marks]

When working with matrices we often need to find the determinant of the matrix.

Prove by the method of mathematical induction that det (An) = (det A)n for n = 1, 2, 3, …

NOTE: det (A x B) = det (A) x det (B)

**3.** [7 marks]

Use the principles of Mathematical Induction to prove that these results are true for all positive integers *n*.

1 + 4 + 7 + ... + (3*n* – 2) = 

**4.** [8 marks]

Use the principles of Mathematical Induction to prove that these results are true for all positive integers *n*.

*n*3 + 2*n* is always a multiple of 3

**5.** [8 marks]

Use the principles of Mathematical Induction to prove that these results are true for all positive integers *n*.

3 + 6 + 12 + 24 + … + (3 x 2n-1) = 3(2n – 1)

**6.** [8 marks]

Use the principles of Mathematical Induction to prove that these results are true for all positive integers *n*.

1 + 2cos x + 2cos 2x + 2cos 3x + … 2cos nx =

Hint: 2(cos A)(sin B) = sin (A + B) – sin (A – B)